Coherence-preserving chaos in a mixed quantum classical description

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We give here a physical picture that exponential instability can occur with the coherence of states preserved in a mixed quantum classical description. Taking advantage of the concept of the geometrical structure of a quantum system, we are able to illustrate our ideas with some examples in a simple manner. This kind of chaos may be important for further investigations of chaotic behaviors in systems with both quantum and classical degrees of freedom.

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I. INTRODUCTION

Chaotic behaviors in classical systems are well understood now [1]. However, whether chaos in the strict meaning, i.e., exponential instability, can occur in quantum systems or not is far from clear [2,3]. It is argued that there is no exponential instability in autonomous bounded quantum systems [4,5]. Recent studies in the field of quantum chaos also show that usually quantum fluctuation and interference tend to suppress classical chaos [6–8]. These may be the reasons why the current research of quantum chaos is mainly focused on the classical quantum correspondence in the semiclassical regime [2]. While it is still vague how the classical chaos manifests itself in the properties of a corresponding full-quantized system [3], another avenue of quantum chaos research derives from the observation that for some systems of theoretical and practical importance the system divides in a natural way into two interacting subsystems, one of which is treated quantum mechanically, whereas the other is treated in the frame of classical mechanics [9,10]. It is shown that for those dynamical systems with both quantum and classical degrees of freedom it may be impossible to make long term forecasts even for the quantum mechanical probabilities. In fact, this phenomenon is not completely new; several similar cases, though not treated so seriously in this sense, have already been reported, for example, a two-level system interacting with the electromagnetic field of a laser cavity [11] and the nuclear collective motion [12]. As one of the main aims of the article, we will analyze the above-mentioned systems from a rather fundamental point of view, which may push the research of the so called semiquantum chaos forward to a certain extent and help to find more analogous systems solvable in a simple manner.

Since Schrödinger first proposed the idea of what are now called "coherent states" in order to show quantum classical correspondence, developments in the field of coherent states and their applications have been breathtaking [13]. The theory of coherent states is taking on increasing importance in the attempt to understand the fundamental aspects of quantum mechanics. The coherent state theory has also been a useful tool for the research of quantum chaos [14,15]. Here we try to use it for the research of chaos in mixed quantum classical description. The geometry on the space of parameters that label the coherent states induced by the group structure is a complex homogeneous space with a natural symplectic structure and can be looked at as a "curved" phase space of the quantum system [16,17]. Because the algebraic structure of operators in the quantum phase space is preserved if the operators are linear functions of the generators of the dynamical group, we find a variety of systems containing both quantum and classical degrees of freedom, whose dynamical evolution can be completely determined by a set of classical-like canonical equations.

When the Hamiltonian of a quantum system is a linear function of the generators of its dynamical group, the system is completely integrable, having a kind of dynamical symmetry [18]. If this quantum system is coupled with a classical system, the structure of the phase space can be destroyed by the interaction. But this interaction can keep the coherence of states simultaneously. Previous researches on the quantum manifestations of classical chaos, which show that the uncertainty of a wave packet initiated at a coherent state will exponentially increase within a scope of time as the correspondence of classical instability [19,20], may give us a mistaken impression that keeping the coherence of states cannot be realized together with the chaotic behaviors.

This article is organized as follows. In Sec. II, we present a brief review of the geometrical structure of a quantum system. This will greatly simplify our discussions in the next sections. In Sec. III, after some general considerations, examples of quantum systems with the dynamical groups SU(2), H_4, SU(1,1), and H_6, respectively, are shown to be possible to have exponential instability with coherence preserved when they are coupled with classical degrees of freedom. In Sec. IV, we summarize our results and present some discussions.

II. GEOMETRICAL STRUCTURE OF A QUANTUM SYSTEM

It is well known that for an arbitrary quantum system, there always exists a group structure. The Hamiltonian H and the elementary transition operators T can be expressed as functions of a set of self-adjoint operators (T_i,

$i=1,2,\ldots$). The operators T_i form an algebra g: $[T_i,T_j]=\sum_{k=1}^{a}C_{ij}^{k}T_k$ where the coefficients C_{ij}^{k} are the structure constants of g. The dynamical group denoted by G is the covering group of g defined by a unitary mapping

$$g\rightarrow G=\exp\left[i\sum_j \alpha_j T_j\right] . \quad (1)$$

According to the group representation theory, the state space as a representation space of G can be decomposed into a direct sum of the various unitary irreducible representation (irrep) carrier spaces of g. So the description of a quantum system can be confined to a certain irrep of G. For the harmonic oscillator, $H=\hbar\omega(a^\dagger a+1/2)$. The dynamical algebra g is the well known Heisenberg-Weyl algebra H_4. Its basis has the following structure:

$$\begin{aligned}&[a,a^\dagger]=I,\quad [\hat{n},a^\dagger]=a^\dagger,\quad [\hat{n},a]=-a ,\\ &[a,I]=0,\quad [a^\dagger,I]=0,\quad [\hat{n},I]=0 .\end{aligned} \quad (2)$$

The corresponding Hilbert space is the Fock space $V^F(|0\rangle,|1\rangle,\ldots)$. The basis states are specified by the particle number operator $\hat{n}$, i.e., the complete set of commutable observables (CSCO) of H_4. For the spin system, the basic operators are (J_+,J_-,J_0) with the relations

$$[J_+,J_-]=2J_0,\quad [J_0,J_-]=-J_-,\quad [J_0,J_+]=J_+ . \quad (3)$$

These operators span the su(2) algebra. The Hilbert space is the $(2j+1)$-dimensional irrep space V^{2j+1} of su(2). The basis states $|jm\rangle$ are specified by the CSCO (J^2,J_0) of the subgroup chain: su(2)$\supset$u(1) through the equations

$$J^2|jm\rangle=j(j+1)|jm\rangle,\quad J_0|jm\rangle=m|jm\rangle . \quad (4)$$

For the one-dimensional hydrogen atom, $H=(p^2/2)-(1/r)$. The dynamical algebra is the su(1,1) algebra whose generators obey

$$[K_1,K_2]=-iK_3,\quad [K_2,K_3]=-iK_1,\quad [K_3,K_1]=iK_2 . \quad (5)$$

Its discrete irrep $D^\dagger(k)$ has the basis states ($|kn\rangle$, $n=0,1,\ldots,\ k>0$) which are eigenvectors of CSCO's (K^2,K_3) of the subgroup chain: su(1,1)$\supset$u(1) where $K^2=K_3^2-K_1^2-K_2^2$ and K_3 satisfy

$$K^2|kn\rangle=k(k-1)|kn\rangle,\quad K_3|kn\rangle=(k+n)|kn\rangle . \quad (6)$$

Generally, for a dynamical Lie algebra of l rank and n dimensional, there is a CSCO (Q_i) [21]:

$$[Q_i,Q_j]=0,\quad i,j=1,2,\ldots,d,\quad d=\frac{n-l}{2}+l . \quad (7)$$

Because l operators of this set specify the irrep carrier space of g or the Hilbert space, there are only $(n-l)/2$ operators which are non-fully-degenerate. The basis states of the irrep can be specified by $(n-l)/2$ quantum numbers. These operators completely determine the structure of the Hilbert space.

If we fix a state $|\psi_0\rangle$ in the Hilbert space, then all the states $|\psi\rangle$ of the system can be generated by $(n-l)/2$ elementary excitation operators $(X_i^\dagger)$ as follows:

$$|\psi\rangle=F(X_i^\dagger)|\psi_0\rangle . \quad (8)$$

Suppose S is the maximal stability subgroup of G with respect to the fixed state $|\psi_0\rangle$. Then G/S is the basic geometrical manifold of the state space. We can easily see this from the associated generalized coherent states of G [16]:

$$\begin{aligned}|\Omega\rangle&\equiv\exp\left[\sum_{i=1}^{(n-l)/2}(z_iX_i^\dagger-\text{H.c.})\right]|\psi_0\rangle\\ &=K^{-1/2}(z,z^*)\exp\left[\sum_{i=1}^{(n-l)/2}(z_iX_i^\dagger)\right]|\psi_0\rangle . \end{aligned} \quad (9)$$

The normalization constant $K(z,z^*)$ is known as the Bergmann kernel [22]. Then according to differential geometry theory [23], there exists a closed nondegenerate two-form ω on G/S whose explicit form in the complex local system is

$$\begin{aligned}\omega&=i\hbar\sum_{ij}\frac{\partial^2\ln K(z,z^*)}{\partial z_i\partial z_j^*}dz_i\wedge dz_j^*\\ &\equiv i\hbar\sum_{ij}g_{ij}dz_i\wedge dz_j^*\end{aligned} \quad (10)$$

and the corresponding Poisson bracket is

$$[f,g]_p=\frac{1}{i\hbar}\sum_{ij}g^{ij}\left[\frac{\partial f}{\partial z_i}\frac{\partial g}{\partial z_j^*}-\frac{\partial g}{\partial z_i}\frac{\partial f}{\partial z_j^*}\right] . \quad (11)$$

By introducing the canonical coordinates (q,p) on G/S, the Poisson bracket takes the standard form

$$[f,g]_p=\sum_i\left[\frac{\partial f}{\partial q_i}\frac{\partial g}{\partial p_i}-\frac{\partial g}{\partial q_i}\frac{\partial f}{\partial p_i}\right] . \quad (12)$$

Any operator can be expressed in the so called Q symbol or phase space representation:

$$A\rightarrow A(z,z^*)=\langle\Omega|A|\Omega\rangle . \quad (13)$$

For the H_4 case, the coherent states are

$$|\Omega\rangle=\exp(za^\dagger-z^*a)|0\rangle,\quad K(z,z^*)=\exp(zz^*) . \quad (14)$$

The canonical position and momentum coordinates satisfy

$$z=\frac{1}{\sqrt{2}}(q+ip),\quad z^*=\frac{1}{\sqrt{2}}(q-ip) . \quad (15)$$

In this representation, the Q symbols of the operators $a^\dagger a$, $(a+a^\dagger)/\sqrt{2}$, and $(a-a^\dagger)/i\sqrt{2}$ are $(p^2+q^2)/2$, q, and p, respectively. In the su(2) case, coherent sates are

$$\begin{aligned}&|j\Omega\rangle=\exp(zJ_+-z^*J_-)|j,-j\rangle ,\\ &K(z,z^*)=(1+zz^*)^{2j} .\end{aligned} \quad (16)$$

After introducing the local canonical coordinates

$$\frac{1}{\sqrt{4j}}(q+ip)=\frac{z}{\sqrt{1+zz^*}} \quad (17)$$

we can transform the Poisson bracket into the standard form. The phase space representations of the generators are

$$J_+(q,p)=\langle j\Omega|J_+|j\Omega\rangle$$
$$=\frac{2jz^*}{1+zz^*}$$
$$=1/2(q-ip)\sqrt{4j-p^2-q^2}\ , \quad (18)$$

$$J_-(q,p)=J_+^*(q,p)\ , \quad (19)$$

$$J_0(q,p)=\langle j\Omega|J_0|j\Omega\rangle$$
$$=j\frac{zz^*-1}{1+zz^*}$$
$$=1/2(p^2+q^2-2j)\ . \quad (20)$$

In the su(1,1) case, coherent states are

$$|k\Omega\rangle=\exp(zK_+-z^*K_-)|k,0\rangle\ ,$$
$$K_+=K_1+iK_2\ , \quad (21)$$
$$K(z,z^*)=(1-zz^*)^{-2k}\ .$$

The canonical local coordinates q,p take the form

$$\frac{1}{\sqrt{4k}}(q+ip)=\frac{z}{\sqrt{1-zz^*}}\ . \quad (22)$$

The phase space representations of the generators are

$$K_+(q,p)=\langle k\Omega|K_+|k\Omega\rangle$$
$$=\frac{2kz^*}{1-zz^*}$$
$$=1/2(q-ip)\sqrt{4k+p^2+q^2}\ , \quad (23)$$

$$K_-(q,p)=K_+^*(q,p)\ , \quad (24)$$

$$K_3(q,p)=\langle k\Omega|K_3|k\Omega\rangle$$
$$=k\frac{1+zz^*}{1-zz^*}=\left[k+\frac{p^2+q^2}{2}\right]\ . \quad (25)$$

For detailed information about the Q symbol in the representation of canonical coordinates, especially in the cases of other dynamical groups, the reader may refer to papers [24,25].

III. COHERENCE-PRESERVING CHAOS

In Sec. II, based on the basic structure of quantum mechanics, we introduced the concept of quantum phase space. In the quantum phase space, provided there are suitable local coordinates on the manifold G/S, a unique and explicit phase space representation of any quantum state or operator can be given. But we have not touched the dynamics of quantum mechanics. Generally, for any two arbitrary operators A and B,

$$\langle\Omega|[A,B]|\Omega\rangle\neq i\hbar[A(p,q),B(p,q)]_p\ , \quad (26)$$

which means the algebraic structure between operators is usually not preserved in the phase space representation due to the quantum fluctuations manifested in the quantum phase space. This is just the elementary difference between quantum and classical mechanics.

However, the commutation relations for generators of the dynamical group do survive [26]:

$$\langle\Omega|[T_i,T_j]|\Omega\rangle=i\hbar[T_i(p,q),T_j(p,q)]_p\ . \quad (27)$$

This property can be easily verified using previous examples. So only if operators are linear functions of the generators can the algebraic structure be preserved. Now we consider the following system:

$$H=\sum_i f_i(P,Q)T_i+f_0(P,Q)\ , \quad (28)$$

where P and Q are vectors in the phase space of the classical subsystem and T_i are generators of the dynamical group of the quantum subsystem. So the above Hamiltonian represents a variety of systems in a mixed quantum classical description. A state of the system can be denoted as $(P,Q,|t\rangle)$ where $|t\rangle$ is the state of the quantum subsystem. The dynamical evolution of the classical variables P,Q is determined by the equations:

$$\frac{dP}{dt}=\left\langle t\left|\frac{\partial H}{\partial Q}\right|t\right\rangle\ , \quad (29)$$

$$\frac{dQ}{dt}=-\left\langle t\left|\frac{\partial H}{\partial P}\right|t\right\rangle\ . \quad (30)$$

For the quantum variable T_i^H in the Heisenberg picture, we have

$$T_i^H=U^{-1}(t,t_0)T_iU(t,t_0)\ , \quad (31)$$

$$\frac{dT_i^H}{dt}=\frac{1}{i\hbar}[T_i^H,H^H]\ . \quad (32)$$

Suppose $t_i=t_0+i(t-t_0)/N$, $i=1,2,\ldots,N$, then the time evolution operator $U(t,t_0)$ can be written as

$$U(t,t_0)=\lim_{N\to\infty}\prod_{i=1}^{i=N}U(t_i,t_{i-1})\ , \quad (33)$$

where

$$U(t_i,t_{i-1})=\exp\left[-\frac{i}{\hbar}\frac{t-t_0}{N}\left[\sum_j f_j(P(t_i),Q(t_i))T_j+f_0(P(t_i),Q(t_i))\right]\right]\ . \quad (34)$$

For H is only a linear function of the generators of the dynamical group, the operator $U(t,t_0)$ is just a group element $g(t,t_0)\in G$ with an unimportant phase factor ignored. We can derive this result directly from the above equations (1), (33), and (34).

Using the basic theorem of group theory, one can find the following decomposition is unique [27]:

$$g(t,t_0)\Omega=\Omega'(t)s(t),\quad \Omega,\Omega'\in G/S;\quad s\in S\ . \quad (35)$$

Then we have

$$U(t,t_0)|\Omega\rangle = g(t,t_0)\Omega|\psi_0\rangle = \Omega'(t)s(t)|\psi_0\rangle = |\Omega'(t)\rangle e^{i\phi(t)} . \tag{36}$$

In getting this relation, the property of the operator s as an element of the maximum stability subgroup S with respect to $|\psi_0\rangle$ has been used. Equation (36) just means that a coherent state will be always a coherent one during its evolution. As a result of Eqs. (27) and (36),

$$\left\langle\Omega\left|\frac{dT_i^H}{dt}\right|\Omega\right\rangle = \frac{d}{dt}\langle\Omega|T_i^H|\Omega\rangle = \frac{d}{dt}T_i(p(t),q(t)) , \tag{37}$$

$$\frac{1}{i\hbar}\langle\Omega|[T_i^H,H^H]|\Omega\rangle = \frac{1}{i\hbar}\langle\Omega'(t)|[T_i,H]|\Omega'(t)\rangle = [T_i(p(t),q(t)),H_{\text{eff}}(p(t),q(t),P(t),Q(t))]_p , \tag{38}$$

The above expression $H_{\text{eff}}(p,q,P,Q)$ is defined as follows:

$$H_{\text{eff}}(p(t),q(t),P,Q) = \langle\Omega'(t)|H|\Omega'(t)\rangle . \tag{39}$$

So after taking the Q symbol of Eq. (32), we immediately have

$$\frac{d}{dt}T_i(p(t),q(t)) = [T_i(p(t),q(t)),H_{\text{eff}}(p(t),q(t),P(t),Q(t))]_p , \tag{40}$$

Combining Eqs. (29), (30), and (40), we conclude that the exact solutions of the system H are completely determined by H_{eff} through a set of classical-like canonical equations;

$$\frac{dP}{dt} = \frac{\partial H_{\text{eff}}}{\partial Q};\quad \frac{dQ}{dt} = -\frac{\partial H_{\text{eff}}}{\partial P} , \tag{41}$$

$$\frac{dp}{dt} = \frac{\partial H_{\text{eff}}}{\partial q};\quad \frac{dq}{dt} = -\frac{\partial H_{\text{eff}}}{\partial p} . \tag{42}$$

Just in this meaning, we call H_{eff} the effective Hamiltonian for this system. It is not difficult now to verify whether chaos can occur or not. But remember, coherence of states is always preserved in the kind of system expressed by (28). Now we turn to some applications.

A. Example 1: A spin-boson system

The Hamiltonian of this system in the full quantized form is

$$H = b^\dagger b + \omega_0 S_z + \lambda(b+b^\dagger)(S_+ + S_-) . \tag{43}$$

It describes a two-level system interacting with a single mode field ignoring the two-photon processes. Before its full quantized behaviors were investigated carefully to understand quantum chaos [28,29], many papers dealing with the system semiclassically had been published [30,31]. Recently, in the consideration of nuclear collective motion [12], the same method treating the field as a classical variable was applied. Here we present our way in a systematic way and a simple manner.

(i) No field quantization. This Hamiltonian is rewritten into

$$H^{(1a)} = \tfrac{1}{2}(P^2+Q^2) + \omega_0 S_z + \sqrt{2}\lambda Q(S_+ + S_-) . \tag{44}$$

The quantized subsystem posseses an SU(2) dynamical group. $H^{(1a)}$ is a linear function of the generators. So the dynamics can be determined by the effective Hamiltonian

$$H_{\text{eff}}^{(1a)}(p,q,P,Q) = \tfrac{1}{2}(P^2+Q^2) + \frac{\omega_0}{2}(p^2+q^2-2s) + \sqrt{2}\lambda qQ\sqrt{4s-p^2-q^2} . \tag{45}$$

The behavior of the trajectory $(p(t),q(t),P(t),Q(t))$ in the four-dimensional phase space can be chaotic due to previous papers [29,31].

(ii) Only field quantization. As far as we are aware, this case has not been touched. Here we will not discuss under what conditions we can treat the spin as a classical degree of freedom while the field quantization is important. Our interest lies only in the dynamical behaviors in the mixed quantum classical description. The Hamiltonian in this case is

$$\begin{aligned} H^{(1b)} &= b^\dagger b + \omega_0 S_z + 2\lambda(b+b^\dagger)S_x \\ &= b^\dagger b + \frac{\omega_0}{2}(P^2+Q^2-2s) \\ &\quad + \lambda Q\sqrt{4s-P^2-Q^2}(b+b^\dagger) , \end{aligned} \tag{46}$$

in which the classical canonical variables P and Q satisfying

$$S_z = \tfrac{1}{2}(P^2+Q^2-2s),\quad S_x = \sqrt{s^2-S_z^2}\frac{Q}{\sqrt{P^2+Q^2}} , \tag{47}$$

have been introduced. Obviously $H^{(1b)}$ is a linear function of generators of the group H_4. Then the effective Hamiltonian is

$$H^{(1b)}(p,q,P,Q) = \tfrac{1}{2}(p^2+q^2) + \frac{\omega_0}{2}(P^2+Q^2-2s) + \sqrt{2}\lambda Qq\sqrt{4s-P^2-Q^2} . \tag{48}$$

Interestingly, we find

$$H_{\text{eff}}^{(1a)}(p,q,P,Q) = H_{\text{eff}}^{(1b)}(P,Q,p,q) . \tag{49}$$

So the structure of the phase space of $H_{\text{eff}}^{(1b)}$ is identical with that of $H_{\text{eff}}^{(1a)}$. The coherent states of H_4 are found first and known best. It has the minimum uncertainty:

$$\langle \Delta Q^2 \rangle = \langle \Delta P^2 \rangle = \frac{\hbar^2}{4} . \tag{50}$$

We should emphasize here the physical picture: A minimum uncertainty wave packet evolves chaotically, in the meaning of the exponential instability of its average position and average momentum, but its width or its coherence is preserved all the time. This unusual feature may exist only in the mixed quantum classical description.

B. Example 2: A classical oscillator interacting with a purely quantum mechanical oscillator

The Hamiltonian is [10]

$$H^{(2)} = \tfrac{1}{2}(\hat{p}^2 + \hat{q}^2 + P^2 + \hat{q}^2 Q^2) . \tag{51}$$

It can be also written as

$$H^{(2)} = a^\dagger a + \tfrac{1}{2}P^2 + \frac{a^2 + a^{\dagger 2} + 2a^\dagger a + 1}{4} Q^2 . \tag{52}$$

We define the three operators

$$K_1 = \frac{a^{\dagger 2} + a^2}{4}, \quad K_2 = \frac{a^{\dagger 2} - a^2}{4i}, \quad K_3 = \frac{a^\dagger a + \frac{1}{2}}{2} . \tag{53}$$

It is easy to verify K_1, K_2, and K_3 satisfy the commutation relations in (5). So the three operators span the su(1,1) algebra. Noticing that

$$H^{(2)} = (Q^2 + 2)K_3 + Q^2 K_1 + \tfrac{1}{2}P^2 + \text{const} \tag{54}$$

as a linear function of the generators, we can describe the system by an effective Hamiltonian. In the Fock space V^F, a simple calculation tells us

$$K^2 = K_3^2 - K_1^2 - K_2^2 = -\tfrac{3}{16} , \tag{55}$$

which means the Hilbert space is the irrep carrier space of su(1,1) denoted as $D^\dagger(\frac{1}{4})$. The effective Hamiltonian is found out easily based on our preparations:

$$H_{\text{eff}}^{(2)}(p,q,P,Q) = (Q^2 + 2)\left[\frac{1}{4} + \frac{p^2 + q^2}{2}\right] + \frac{P^2}{2} + \frac{Q^2}{2} q \sqrt{1 + p^2 + q^2} . \tag{56}$$

In fact, our present work was induced by the very recent paper [10] by Cooper *et al.* In their paper, it seems that a different-looking effective Hamiltonian was obtained by accident. And the phase space was introduced unnaturally. But we would like to compare our results to theirs. After defining the quantity

$$G = \langle \Omega | \hat{q}^2 | \Omega \rangle = 2K_1(q,p) + 2K_3(q,p) \tag{57}$$

we immediately obtain the relation appearing in the paper [10]:

$$\frac{1}{2}\frac{d^2G/dt^2}{G} - \frac{1}{4}\left[\frac{dG/dt}{G}\right]^2 - \frac{1}{4G^2} + 1 = 0 . \tag{58}$$

The Gaussian wave packets they used are just the coherent states of su(1,1), or the so called restricted kind of squeezed states. Their numerical results assure us chaos can appear in this system preserving the generalized coherence of the su(1,1) algebra.

C. Example 3: Two-photon processes derived by a classical source

The Hamiltonian takes the form

$$H = \hbar[f_1(t) a^\dagger a + f_2(t) a^2 + f_2^*(t) a^{\dagger 2} + f_3(t) a + f_3^*(t) a^\dagger] . \tag{59}$$

The dynamical algebra is H_6 made up of the six operators $(a^\dagger a + 1/2, a^{\dagger 2}, a^2, a^\dagger, a, I)$. As we can find, H is again a linear function of the generators of the dynamical group when we enlarge it from H_4 to H_6. Detailed information related to the construction and the application of the coherent states in this case can be found in the Ref. [32]. Here our aim is to point out that, in this system, coherence-preserving chaos can also occur. So we would like to discuss it in a different way. Yuen [33] has already calculated the explicit expression for the time evolution operator in the representation of the coherent states of H_4:

$$\langle \alpha | U(t,t_0) | \beta \rangle = \exp\{-1/2|\alpha|^2 - 1/2|\beta|^2 + A(t) + B(t)\beta^2 + C(t)\alpha^{*2} + [D(t)+1]\alpha^*\beta + E(t)\beta + F(t)\alpha^*\} , \tag{60}$$

where $A(t)$, $B(t)$, $C(t)$, $D(t)$, and $F(t)$ satisfy a set of differential equations dependent on the functions $f_1(t)$, $f_2(t)$, and $f_3(t)$:

$$\frac{dA}{dt} = -2(2f_2^* C + f_2^* F^2 + f_3^* F), \quad \frac{dB}{dt} = -i f_2^* (D+1)^2 , \tag{61}$$

$$\frac{dC}{dt} = -2(4f_2^* C^2 + 2f_1 C + f_2), \quad \frac{dD}{dt} = -i(4f_2^* C + f_1)(D+1) , \tag{62}$$

$$\frac{dE}{dt} = -i(2f_2^* F + f_3^*)(D+1), \quad \frac{dF}{dt} = -i[(4f_2^* C + f_1)F + 2f_3^* C + f_3^*] . \tag{63}$$

The above set of equations are integrable. But if f_1, f_2, and f_3 are not known functions of t but determined by the interactions between the classical field and the quantum subsystem like

$$f_i = f_i(a^\dagger(t), a(t)) = f_i(A(t), B(t), C(t), D(t), E(t), F(t)) \tag{64}$$

there are enough reasons to expect nonintegrability to appear as in the previous examples. Then the time evolution operator $U(t,t_0)$ will show the very sensitivity on initial conditions. But the generalized coherence of H_6 is still preserved for the simplicity of the Hamiltonian.

IV. CONCLUSION AND DISCUSSION

After introducing the concept of quantum phase space, i.e., the geometrical structure of a quantum system, we are able to analyze a variety of systems in a mixed quantum classical description conveniently. It is found that chaos in its strict meaning can occur when quantum degrees of freedom are coupled with classical degrees of freedom. Examples in which the Hamiltonian is expressed as a linear function of the generators of the dynamical group belonging to the quantum subsystem are demonstrated to be possible to show chaotic behaviors with coherence of states preserved. These systems can be described by a classical-like effective Hamiltonian without any approximation.

As every quantum system has a group structure, our method may have some applications in other systems. The dynamical groups U(N), SO($2N$), and SP($2N$) have been extensively used to describe both the atomic and nuclear systems. The concept of quantum phase space is of importance to understanding the correspondence of quantum and classical mechanics in these systems. When these systems are coupled with classical degrees of freedom, we say it is possible for exponential instability to occur at least when the Hamiltonians are linear functions of generators of their corresponding dynamical groups of the quantum subsystems. When the Hamiltonian contains small parts expressed as nonlinear functions of the generators of the dynamical group of the quantum subsystem, we conjecture that the exponential instability may not disappear because the complexity of the phase space structure may not change much in the chaotic region while subjected to a small perturbation. This point will help to understand semiquantum chaos in a larger number of systems. Coherence-preserving chaos gives us a different physical picture, and applications may be found, particularly in the understanding of the irregular behaviors in the full quantized systems. What are other kinds of chaos, i.e., exponential instability, in the mixed quantum classical description, and how can they be described in a systematic way, are both interesting questions.

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